Rotation Between Two Vectors in $\mathbb{R}^3$

This is valid and numerically stable as long as the two vectors $a$ and $b$ are not pointing in opposite directions:

$$a, b \in \mathbb{R}^3$$
$$\hat{a} = \frac{a}{\|a\|}$$
$$\hat{b} = \frac{b}{\|b\|}$$
$$\omega = \hat{a} \times \hat{b}$$
$$c = \frac{1}{1 + \hat{a}^T \hat{b}}$$

$$bR_a = I_3 + \omega \times + c\omega \times$$

$$= \begin{pmatrix}
1 - c (\omega_2^2 + \omega_3^2) & c\omega_1\omega_2 - \omega_3 & c\omega_1\omega_3 + \omega_2 \\
c\omega_1\omega_2 + \omega_3 & 1 - c (\omega_1^2 + \omega_3^2) & c\omega_2\omega_3 - \omega_1 \\
c\omega_1\omega_3 - \omega_2 & c\omega_2\omega_3 + \omega_1 & 1 - c (\omega_1^2 + \omega_2^2)
\end{pmatrix}$$

When $\hat{a}^T \hat{b} < \epsilon - 1$, the rotation is best parameterized by first flipping around a vector perpendicular to $a$. Let such a vector be $p$:

$$p^T a = 0$$
$$p^T p = 1$$

The rotation which leaves $p$ fixed and sends $\hat{a}$ to $-\hat{a}$ is then:

$$-a R_a = 2pp^T - I_3$$

Composing this with $bR_{-a}$ yields the desired rotation, avoiding singularities:

$$bR_a = bR_{-a} \cdot -a R_a$$

$$= \left( I_3 - \omega \times + \frac{1}{1 - \hat{a}^T \hat{b}} \cdot \omega \times \right) \cdot (2pp^T - I_3)$$